Semi-convection and overshooting in intermediate-mass and massive stars

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Abstract. Two mixing schemes, overshooting and semi-convection, are studied in the framework of evolutionary models for Pop. I intermediate-mass and massive stars up to 20M☉. Overshooting is applied either to the core boundary, or to the boundaries of all convective zones. A generalized computational algorithm is proposed for semi-convection. The evolutionary models are computed from the quasi-hydrostatic Pre-Main-Sequence phase to the end of core helium burning. They are compared to models adopting the standard mixing scheme with no mixing beyond the Schwarzschild’s limit. Overshooting, when applied to the boundaries of all convective zones, does not modify the evolutionary features of the stars. We find that semi-convection does not develop in intermediate-mass stars. In contrast, semi-convection and core overshooting completely modify the structural evolution of massive stars after central hydrogen exhaustion. The tracks followed in the H-R diagram and the times spent in different phases of evolution disfavour the models with core overshooting and favours the ones including semi-convection.

Key words: stars: interiors – stars: evolution – convection

1. Introduction

Mixing of matter takes place at various levels inside the stars at almost all phases of their evolution, and is of crucial importance for their structural and chemical evolution. Its theoretical study and numerical treatment is indispensable to a better understanding of those objects. The mixing length theory developed by Böhm-Vitense (1958) gives an attractive simple approach to treat convectively unstable regions. It helped understanding the main features in the evolution of the stars, and is still used in most stellar evolutionary codes. However, observationally and theoretically, it appears necessary to overcome some of the shortcomings related to the basic assumptions and to the local character of the mixing length theory. This is why overshooting and semi-convection have been introduced in the modeling of stars.

Overshooting is associated with the use of the Schwarzschild criterion, which locates the borders of the convective zones at the point where the convective bubbles are no more subjected to the buoyancy force. At this point, the bubbles have non zero velocity, and continue to move on some distance before coming to rest. The study of this convective penetration from the core boundary, or core overshooting, into the stable radiative layers has led to contradictory conclusions. The predicted extent of overshooting varies from being negligible to almost doubling the mass of convective cores (e.g. Shaviv & Salpeter 1973; Maeder 1975; Roxburg 1978; Bressan et al. 1981; Langer 1986; Zahn 1991). The confrontation of evolutionary models including core overshooting with observations did not lead to commonly accepted conclusions either (Mermilliod & Meynet 1986; Maeder & Meynet 1989; Stothers 1991; Napiwotzki et al. 1991; Lattanzio et al. 1991; Alongi et al. 1991). Convective envelope overshooting, on the other hand, is known to affect the evolutionary features of the stars during their core helium burning phase (Stothers & Chin 1991; Alongi et al. 1991).

Semi-convection refers to the partial mixing between adjacent layers of non-homogeneous chemical composition that is required in order to obtain a coherent solution of the internal structure of the star. This phenomenon was first encountered by Schwarzschild & Härm (1958) in the context of main sequence massive stars. Its impact on the structural evolution of these stars has then been analysed by several authors, e.g. Stothers (1965, 1970), Chiosi & Summa (1970), Eggleton (1972), Sreenivasan & Ziebarth (1974). The development of semi-convective zones was also discovered to take place during helium burning in low-mass stars (Schwarzschild & Härm 1969), and Paczyński (1970) found the phenomenon to be present during the same evolutionary phase in intermediate-mass stars as well. Robertson & Faulkner (1972) gave a simplified numerical treatment of it.

The modifications in the chemical abundance profiles brought by overshooting and semi-convection affect the internal structure of the stars in various ways. Core overshooting,
by bringing fresh elements from the layers outside the former boundary, results in more massive convective cores during the central burning phases. The models run at higher luminosities, the stars spend more time burning their central hydrogen, while the helium burning phase is shortened (Bertelli et al. 1985). Semi-convection has the same effects when it develops at the border of the convective core. This can be the case in main sequence massive stars (but Langer et al. 1985, find that the semiconvective zones do not always come in contact with the convective core; see also the conclusions of this study), or during core helium burning for low- and intermediate-mass stars (but Lattanzio et al. 1991, find very little or no semi-convection for stars more massive than 4 M☉). In massive stars, the most important consequences of semi-convection come from its development in intermediate layers of strongly inhomogeneous composition after central hydrogen exhaustion. Mixing in these regions changes the chemical composition of layers into which the hydrogen burning shell enters, being in part responsible of whether or not the star has established its thermal equilibrium as a red supergiant when core helium burning begins (Simpson 1971; Chin & Stothers 1990). Finally, the extent of downward penetration of the convective envelope which develops when the star reaches lower effective temperatures sensitively affects the time spent as a blue supergiant during core helium burning (Robertson 1972; Stothers & Chin 1975) and the extent of blue loops during that phase (Stothers & Chin 1991; Alongi et al. 1991).

Although several results are found in the literature concerning overshooting and semi-convection, very few studies of these two phenomena using a single code have been performed until now (Chin & Stothers 1990), and none with up-to-date input physics. Besides, no study considering overshooting at the boundaries of all convective zones has been performed. This paper presents such calculations for pop I intermediate-mass and massive stars from 2.5 to 20 M☉. Our evolutionary code is described in Sect. 2. A critical analysis of semi-convection treatments is also presented along with our own prescription. Calculations are carried from the early overall Pre-Main-Sequence contraction phase until central helium exhaustion. The use of homogeneous and consistent input data for all our sequences allows the differential effects to be assigned to the different mixing treatments. Their development and the evolutionary features of our models are analysed in Sect. 3, and some possible confrontations with observations are presented in Sect. 4. Conclusions are drawn in Sect. 5.

2. The evolutionary code

2.1. General features

Our evolutionary code integrates the five hydrodynamic equations of the stellar structure by the Henyey technique. We assume spherical symmetry and adopt Lagrangian coordinates. Our five independent variables are the velocity, radius, density, temperature and luminosity. We neglect the effects of rotation and magnetic fields. The equations of the atmosphere are integrated along with the ones describing the stellar interior. Use of tables of atmospheric models of Bell et al. (1975) is made to get the atmospheric temperature stratification. The ionisation of H, He, C, N and O is followed by solving the system of Saha equations through a Newton-Raphson method for hydrogen and helium, and by an iterative procedure for C, N and O. Electrostatic corrections are taken into account through the Debye–Hückel model. The contribution of electron degeneracy to the pressure and internal energy is evaluated from the generalised Fermi-Dirac integrals. Energy losses through (ν, p) pairs are taken from Itoh et al. (1989). The radiative opacities are interpolated linearly in tables of different chemical compositions (in H, He, C and O). At low temperatures (T < 7500K), they are taken from Neuforge (1993). At higher temperatures, we use the new Rogers & Iglesias (1992) opacities. Mass loss in our intermediate-mass stars (2.5 and 5 M☉) is taken into account by the Reimers (1975) formula with η = 0.33. For the massive stars, (10, 15 and 20 M☉) we use the empirical expressions developed by de Jager et al. (1988). In all our models, the mass losses are applied since the beginning of the Main-Sequence phase. Our nuclear network includes all the reactions implied in hydrogen and helium burning. It is described in detail in Mowlavi et al. (1993).

2.2. Interior mixing

The regions where convective instability develops are determined by the Schwarzschild criterion \( \nabla_{\text{rad}} \geq \nabla_{\text{ad}} \), where the radiative gradient is given by

\[
\nabla_{\text{rad}} = \frac{d\ln T}{d\ln P} \bigg|_{\text{rad}} = \frac{3\kappa LP}{16\pi aT^4(GM_r + r^2 \frac{D_T}{D_T})},
\]

\( M_r \) being the mass contained inside the radius \( r \), the other symbols having their usual meaning. The energy flux transport in these regions is evaluated through the mixing length theory. The free parameter \( \alpha \) (defined as the mean distance normalized to the pressure scale height over which the bubbles travel before losing their identity) is usually supposed to lie between 1 and 2. By fitting evolutionary tracks of red giants in the Hertzsprung-Russell diagram (HRD) to the observed positions of stars in clusters and associations, Maeder & Meynet (1989) find a value of \( \alpha = 1.9 \pm 0.1 \). The same adjustment with the new Rogers & Iglesias (1992) opacities leads to \( \alpha = 1.6 \pm 0.1 \) (Schaller et al. 1991). This value is reasonably close to the ones adopted by other authors. For example Lattanzio (1991) takes \( \alpha = 1.5 \). Boothroyd & Sackman (1988), although adopting a value of \( \alpha = 1 \) in their study of low mass stars up to the AGB phase, suggest a higher value up to \( \alpha = 2 \). Here, we chose \( \alpha = 1.5 \). We further make the assumption of instantaneous mixing in all our convective zones. Finally, let us mention that, in order to define the limits of the convective core accurately, we impose that the layers at its borders contain at most one thousandth of the total stellar mass of the star.
2.2.1. Overshooting

Overshooting has been the subject of many papers during the last decade, and will not be discussed here. We refer to Zahn (1991) and references therein for a comparative study of the current convective penetration prescriptions used for stellar interiors. In our evolutionary sequences including this phenomenon, we simply assume that the distance of penetration of the bubbles into the radiative layers is \( d_{\text{over}} = \alpha_{\text{over}} \times H_p \) where \( H_p \) is the pressure scale height evaluated at the border defined by the Schwarzschild criterion. If necessary, we limit the maximum growth of the convective zones to twice their mass, so that \( d_{\text{over}} = \min(\alpha_{\text{over}} \times H_p, d_{2M_*}) \), where \( d_{2M_*} \) is the distance over which the mass of the convective zones doubles. We use a mild overshooting of \( \alpha_{\text{over}} = 0.20 \).

Two sets of models including overshooting are analysed. The first one applies overshooting to the boundaries of the convective core only. This prescription is equivalent to the one used by Maeder & Meynet (1987, 1989). The second one introduces overshooting at the boundaries of all convective regions, including the convective core, the convective envelope, and all fully convective zones which develop in the intermediate layers of massive stars.

2.2.2. Semi-convection

As described in Sect. 2.2.1, some - even very little - material from the convective region is mixed with the adjacent stable layers. Turbulence or meridional circulation can also contribute to this extra-mixing. After the readjustment of the different quantities of Eq. (1) that are sensitive to the chemical composition, these initially stable layers may or may not become convectively unstable. In the first case, the convective region will absorb them and grow. In the second case, the last unstable layer will define the border of the convective zone.

In some situations, a third possibility can appear, which is the development of a semi-convective zone (SCZ) defined by Schwarzschild & Härm (1958) as "a zone which, initially unstable against convection, becomes neutral against convection by partial mixing with the adjacent layers." This definition is very close to the one given by Simpson (1971).

Several conclusions can be drawn from this definition: (1) As mixing between the layers in a SCZ is not complete, the zone is characterized by non-homogeneous compositions. (2) The change in chemical composition induced by semi-convection directly affects the stability of a layer through the change of the local value of the opacity which appears in Eq. (1). So, the main reason for the development of a SCZ lies in the variation of opacity with composition. (3) Convective mixing takes place to just the degree necessary to maintain convective neutrality throughout the SCZ. Which criterion to use for convective neutrality is actually not clear yet. In a medium with varying chemical composition, Ledoux (1947) showed that the right criterion for the onset of convection must be based on density gradients. The resulting Ledoux criterion reduces to the Schwarzschild criterion in a chemically homogeneous zone. From a study based on a global perturbation theory, Gabriel (1969) confirms that the Ledoux criterion should be used. However Kato (1966) showed that, if the medium is thermally conductive, which is the case in the stellar interiors, a region where \( \nabla_{\text{rad}} < \nabla_{\text{led}} \) is vibrationally unstable (as compared to a dynamically unstable region defined by \( \nabla_{\text{rad}} > \nabla_{\text{led}} \)). Although Kato derived his conclusion from a local analysis, Gabriel (1970) confirmed Kato's results from a completely different study based on the mixing mechanisms taking place in semi-convective zones. Several authors have tried to discriminate between the two criteria by comparing the results of evolutionary calculations with observations. No definite conclusion can be drawn yet. We use the Schwarzschild criterion, i.e. we suppose that mixing is instantaneous even in vibrationally unstable regions.

A proper numerical treatment of semi-convection according to the original definition needs a complex algorithm and is very computer-time consuming. Simpson (1971) used a diffusive treatment, the diffusion coefficient being evaluated numerically for each zone by requiring the composition profiles to satisfy the neutrality condition. Another, widely used, simplified treatment was given by Robertson & Faulkner (1972) in the context of massive stars. They made the approximation that the carbon/oxygen ratio is unchanged by their assumed linear perturbation. Castellani et al. (1985) and Boothroyd & Sackmann (1988) removed this approximation by taking into account the effect of C and O opacities on the radiative gradient. Their procedure is designed for core helium burning and has been extended by Boothroyd & Sackmann (1988) to AGB stars. We generalize the method so that it can be applied to all physical situations leading to the development of a semi-convective zone.

First of all, the convective zones in the star are searched for according to the Schwarzschild criterion. Each of them is then processed in turn through the following procedure. Starting from a homogeneous zone within a convective region, shells are added in a symmetrical way, once from the layers above, once from the layers under the convective region. Let us suppose that an upper adjacent layer \( J \) is added. This layer is mixed in the convective region and the radiative temperature gradient is reevaluated with the new composition over the whole region. If the newly added shell becomes convective, it is included in the convective zone and the procedure goes on with a layer adjacent to the lower boundary. Otherwise, layer \( J \) remains radiative and defines the consistent upper boundary of the convective region. Then the procedure continues only downwards. A third possibility can occur: while the addition of the upper adjacent layer makes it convective, another layer within the convective region, say \( K \), becomes radiative. We are typically facing the development of a semi-convective zone according to the definition. The original convective zone extends now up to layer \( K - 1 \) and the procedure goes on by processing the convective zone \( [K+1, J] \): more layers \((J + 1, \ldots)\) are added one by one and mixed to this zone. As the lower layers \((K + 1, \ldots)\) become convectively neutral, they are excluded from the convective zone at the next step, leaving behind a semi-convective zone where convective neutrality is verified. Either a) the procedure goes on until the
currently added shell becomes also convectively neutral. In this case, the convective shell does not exist anymore and is replaced by an extended purely semi-convective zone. Or b) a currently added shell remains radiative after mixing, defining then a semi-convective zone with a convective one on top of it. The same procedure is then applied to the lower boundary of the original convective zone to find its lower consistent boundary and to check for the presence of semi-convection. The procedure is extended in order to handle the possible occurrence of an undetermined number of adjacent semi-convective zones separated by convective ones.

Several remarks have to be made at this point. (1) We have to emphasize that the semi-convection algorithm searches for consistent boundaries of convective zones, during which the value of \( \nabla_{rad} \) is reevaluated throughout the entire zone each time an adjacent layer is added and mixed to it. It can enlarge slightly the convective zones as compared to the results given by the standard mixing scheme. Evolutionary models using this semi-convection procedure might thus display different features than models calculated with the standard mixing scheme, even if semi-convection actually does not develop (see Sect. 4.1). (2) We make the assumption of instantaneous total/partial mixing in the convective/semi-convective zones. Gabriel & Noels (1978) estimate the timescale of these semi-convective regions to be of the order of \( 10^3 \) to \( 10^4 \) years in a 30 M_☉ star, and Langer et al. (1985) find that the vibrational instability does not lead to complete mixing during the fast phases of evolution such as shell hydrogen burning. Our assumption might thus be incorrect for these phases. However, our models will provide a useful tool to test the consequences of efficient partial mixing in regions where semi-convection develops. The reality must lie between our models including semi-convection and the ones without semi-convection, depending on the efficiency of partial mixing in the semi-convective zones. (3) As already mentioned, such an algorithm is very computer-time consuming and can lead to convergence difficulties. Like Boothroyd & Sackmann (1988), we decide to apply our procedure once per time step, after convergence is achieved for the stellar structure equations.

Some authors consider other definitions of semi-convection. One of these definitions considers that the mixing occurs in regions where \( \nabla_{ad} < \nabla_{rad} < \nabla_{Led} \). This zone, being vibrationally unstable, will have a longer mixing timescale than the dynamically unstable ones. In rapidly evolving phases, the former timescale can reach the same order of magnitude, or be greater than the evolutionary timescale. It is the consequent partial mixing occurring in these vibrationally unstable regions which is referred to as "semi-convection". It can be described by an appropriate diffusion approximation, the diffusion coefficient depending on the mixing timescale. We refer to Langer et al. (1985) for a discussion of the diffusion coefficient used by different authors. Let us illustrate this prescription in Fig. 1a. It displays the radiative temperature gradient profile which can be encountered during central helium burning of low-mass stars. The region \([i_1 - i_2]\) above the convective core is characterized by a dynamically unstable (convective) zone \([l_1 - l_2]\) surrounded by two vibrationally unstable (semi-convective) zones \([i_1 - i_1]\) and \([l_2 - i_2]\). Assuming complete efficiency of semi-convection leads to complete mixing in the region \([i_1 - i_2]\) and corresponds to treating its regions according to the Schwarzschild criterion. Assuming no efficiency corresponds to the adoption of the Ledoux criterion (no mixing in the semi-convective zones). If we consider now semi-convection according to the definition of Schwarzschild & Härm (1958) as described above (let us use the Schwarzschild criterion for convective neutrality), a different structure appears. A possible result is shown in Fig. 1b. It is characterized by an inhomogeneous (semi-convective) zone \([i_1 - k_1]\) surmounted by a small homogeneous (convective) one \([k_1 - k_2]\). The original unstable region defined by the Schwarzschild criterion \([i_1 - i_2]\) is thus now extended to \([i_1 - k_2]\).

Besides the prescriptions described above, based on physical considerations, some authors have proposed other techniques to reproduce the effects of semi-convection. Lattanzio (1986), in studying the evolution of \(1 - 3\) M_☉ AGB stars, uses a technique of overshooting, whose extension is restricted by a maximum allowed mass. The size of the overshooted region is determined through a series of tests comparing the resultant abundance profiles and evolutionary features with the results of Faulkner & Canon (1973) who used the Robertson & Faulkner (1972) semi-convection method. Using a somewhat identical philosophy, Caloi & Mazzitelli (1990) show that a naive algorithm based upon a small and ad hoc amount of convective
overshooting can give rise to evolutionary features and helium profiles for a horizontal branch star consistent with those expected from an adequate treatment of semi-convection. These prescriptions must be considered with caution as they do not rely on physical considerations. We finally have to mention the semi-convection defined by Iben & Renzini (1982). Their evolutionary models of AGB stars show a region in the star where the radiative temperature gradient remains very close to the adiabatic one and oscillates around it. They define this zone as being semi-convection, but apply no special treatment to process it correctly. An equivalent attitude is taken by Brunish & Truran (1982) in their study of massive stars. Clearly, their prescription is equivalent to the neglect of semi-convection.

3. Evolutionary features

Evolutionary models were computed for 2.5, 5, 10, 15 and 20 M\(_{\odot}\) stars. All sequences begin with initial models on or near the Hayashi line during the quasi-hydrostatic star contraction. The chemical composition is \((X,Y,Z)=(0.705,0.275,0.02)\), where \(X\), \(Y\) and \(Z\) represent the mass fractions of hydrogen, helium and the heavier elements. The heavy element abundances are taken from Anders & Grevesse (1989). The evolution of these stars in the HRD during their Pre-Main-Sequence (PMS) phase is shown in Fig. 2. The intermediate mass (2.5, 5 M\(_{\odot}\)) stars are initially completely convective, and develop a radiative core after (240, 100) \(10^3\) yr when the central temperatures reach (2.50, 2.07) \(10^6\) K. The initial models of our massive stars have a radiative core, with a deep convective envelope of 7.8 M\(_{\odot}\) for the 10 M\(_{\odot}\), and thin ones of 1 and 0.04 M\(_{\odot}\) for the 15 and 20 M\(_{\odot}\) respectively.

Apart from the initial deuterium burning, the first energetically important nuclear reactions during the PMS are \(^3\)He (3He, 2p) \(^4\)He and \(^12\)C(p, \(\gamma\)) \(^13\)C. They induce the development of a convective core at (3.4, 0.68, 0.13, 0.046, 0.027) \(10^6\) yr. The positions of the stars in the HRD at the time of deuterium burning and development of the convective core are also displayed in Fig. 2.

For each mass, four post-ZAMS evolutionary sequences are computed up to the end of core helium burning, one with our semi-convection treatment, one with overshooting applied to boundaries of all convective zones, one including core overshooting and one sequence, referred to as the standard case, with none of those extra mixing.

3.1. Main-Sequence evolution

The structural evolution of the 5 M\(_{\odot}\) (representative of the intermediate-mass stars) and 15 M\(_{\odot}\) stars (representative of the massive stars) are displayed in Figs. 4 and 5. As far as the structural evolution up to the end of central helium burning is concerned, the 10 M\(_{\odot}\) sequences display a behavior similar to the intermediate-mass stars.

We first compare the models including semi-convection with the ones using the standard mixing scheme. The shrinking of the convective core during central hydrogen burning leaves behind a region with decreasing hydrogen and increasing helium composition. For stars less massive than about 10 M\(_{\odot}\), semi-convection does not develop, and the models with and without semi-convection are identical. For stars more massive than 10 M\(_{\odot}\), however, the hydrogen-rich layers above the convective core tend to be more unstable than the adjacent helium-rich layers (Schwarzschild & H"{a}rm 1958). Such a situation arises first because the contribution of electron scattering to the total opacity increases as the temperature increases and the density decreases. Second, it is well known that the increased importance of the radiation pressure in massive stars favours convection. We find that semi-convection develops very little in our 10 M\(_{\odot}\) star. It becomes important in our 15 and 20 M\(_{\odot}\) stars, in which small convective zones also develop within the region of varying chemical composition. The total mass of the semi-convective zones increases as hydrogen burns in the core. However, their presence does not affect the characteristics of our main sequence models. The size of the convective core, the lifetime of the MS phase and the evolution in the HRD are similar to the ones of the models computed without semi-convection. This is due to the fact that semi-convection does not mix abundances into the convective core. However, it changes the chemical abundance profiles in the intermediate regions. Some authors (see for example Brunish & Truran, 1982) argued that the development of intermediate fully convective zones (IFCZ) which appear in the standard case can mimick the effects of semi-convection. Although we confirm the occurrence of these IFCZ above the convective core, our 15 M\(_{\odot}\) models (Fig. 5a and 5b) show that they involve less mass than the semi-convective zones. In addition, the resulting abundance profiles are different. We will see in Sect.3.2 the importance of these profiles for the Post-Main-Sequence evolution.

Our models of (2.5, 5, 10, 15, 20) M\(_{\odot}\) including overshooting have (33, 22, 20, 14, 13)% more massive convective cores at ZAMS than the standard models. The resulting increase of the
main sequence lifetime lies between 10 and 20% (see Table 3). This widens the main sequence in the HRD by about \( \delta \log(T_{\text{eff}}) \) = 0.05 and 0.04 for massive and intermediate-mass stars respectively. Besides, the models run at slightly larger luminosities as the central hydrogen burns. The fate of the semi-convective zones when core overshooting is applied has often been raised. Our 15 \( M_\odot \) models (Fig. 5c) show that the resulting increase in luminosity due to the core mass increase suppresses the convective instabilities which otherwise develop in the intermediate inhomogeneous regions above the core. This means that no semi-convection would develop in a 15 \( M_\odot \) star during the main sequence phase in the presence of an overshooting of 0.20 \( H_p \) at the convective core boundary. The same conclusion applies to our 20 \( M_\odot \) star. This suppression of semi-convection during CHB in presence of core overshooting is also confirmed by Bressan et al. (1981) using another prescription of overshooting.

3.2. Post-Main-Sequence evolution

After central hydrogen exhaustion, the stars enter a transition phase during which rapid core contraction heats its central parts until the temperatures reach high enough values to ignite helium. This phase is characterized by the formation of a hydrogen burning shell advancing in the inhomogeneous regions. Apart from these general features, the structural evolution is completely different for the 2.5, 5 and 10 \( M_\odot \) and for the 15 and 20 \( M_\odot \) stars.

\( M > 10 \ M_\odot \)

Models including semi-convection (Fig. 5b) reveal the development of large semi-convective zones covering about 3.5 and 7 \( M_\odot \) in the intermediate layers of the 15 and 20 \( M_\odot \) stars. These zones modify the abundance profiles in a nucleus active region, so that their existence has important energetic consequences. Our models show that the stars evolve toward lower effective temperatures and ignite their central helium when they have already reached the red supergiant phase. At that point, the stars possess a deep convective envelope, bringing nuclearly processed material to the surface (first dredge-up phenomenon). During core helium burning (CHHB), a blue loop (BL) in the HRD shifts the star towards higher effective temperatures where they spend most of their CHHB lifetime (80% for our 15 \( M_\odot \) star).

In absence of semi-convection, the transitional phase before helium ignition displays a different behavior. A large intermediate fully convective zone (IFCZ, of 1.6 and 3.5 \( M_\odot \) in our 15 and 20 \( M_\odot \) stars) develops within the hydrogen burning shell. Fresh hydrogen brought in these regions enhances the energy production, and the corresponding HRD tracks run at higher luminosities than the ones of the models including semi-convection. This prevents the convective envelope from penetrating further inside...
until a large fraction of the helium has already been depleted in the core (25 and 75% in our 15 and 20 $M_\odot$ stars). The stars thus ignite their central helium while still being blue supergiants. They move progressively to lower effective temperatures and spend the rest of their CHB phase as red supergiants. We notice that the IFCZ developing in our 20 $M_\odot$ star remains present during half of the CHB phase.

In the models having experienced core overshooting during the MS phase, an IFCZ appears on top of the hydrogen burning shell for a short time prior to the convective envelope penetration. Its mass is not very large in our 15 $M_\odot$, and the star spends its whole CHB phase as red supergiants. In the 20 $M_\odot$ star, however, the mass of the IFCZ becomes important and is located inside the hydrogen burning shell. This prevents the convective envelope to penetrate further inside, and the star evolves in a way similar to the standard case.

When overshooting is applied to the boundaries of all convective zones, a large IFCZ develops prior to central helium ignition. The evolutionary features of these models are similar to the ones displayed by the standard case.

Fig. 4a–d. Structural evolution of the 5 $M_\odot$ stars with a standard mixing scheme, b semi-convection, c overshooting applied to all convective zones, and d core overshooting. Dashed lines give the location of the maximum energy production in the hydrogen/helium burning shell. Filled regions represent fully convective zones, while regions with dotted lines represent semi-convective zones.
The core potential $\phi_c = M_c/R_c$ is a convenient parameter for describing the evolution of the stars as they move from blue to red (or vice-versa), $M_c$ and $R_c$ being the mass and radius (in solar units) of the core, defined as the hydrogen-exhausted region extending from the centre up to the base of the hydrogen burning shell. Lauterborn et al. (1971) showed that a star lies closer to the Hayashi line in the HRD for higher $\phi_c$, and that a critical potential can be found for each stellar mass above which the star becomes a red (super)giant. For our purpose, we define this critical core potential $\phi_{c,c}$ as the value of $\phi_c$ when the effective temperature reaches $\log(T_{eff}) = 3.6$. In our 15 $M_\odot$ stars, $1.035 \leq \phi_{c,c} \leq 1.055$ (see Fig. 6). After a rapid increase, $\phi_c$ reaches a value dependent on the chemical composition profile in the hydrogen burning shell. In the standard case, this value remains slightly under $\phi_{c,c}$, so that the star is still in the blue when helium ignites at the centre. In models including core overshooting during the main sequence phase, more massive cores are produced, and, when the mass of the IFCZ is not very large, the star begins central helium burning as a red supergiant. When overshooting is applied to the boundaries of all convective zones, however, the development of a large IFCZ after the MS phase keeps $\phi_c$ to the values obtained in the standard case.

The evolution during central helium burning is greatly affected by the history of the internal structure. In the stan-
the star moves to higher effective temperatures. The star then spends the rest of its helium burning life as a blue supergiant. Lauterborn et al. (1971) have already noted the importance of the hydrogen profile, whose influence must be combined with considerations on the core potential in order to better understand the occurrence of blue loops in the HRD. The discontinuity in the hydrogen profile also exists when core overshooting is applied. However, in this case, the high \( \phi_c \) values prevent the star from experiencing a blue loop.

We further note a very interesting result in our 15 and 20 M\(_{\odot}\) models including semi-convection: the star exhausts its central helium content while still being a blue supergiant. These models are relevant to the SN1987A supernova whose progenitor is a blue supergiant with an evaluated initial mass of about 20 M\(_{\odot}\) (e.g. Hillebrandt & Höflich 1989; Arnett et al. 1989). The effect of a decrease in metallicity on our models should still be investigated, since the SN1987A supernova is more metal-deficient than the model stars considered here. Langer (1991) also invokes successfully the development of a moderate semi-convection in order to explain the characteristics of the supernova progenitor.

\( M \leq 10 \, M_{\odot} \)

The case of intermediate-mass stars is much simpler, at least before central helium burning. During that phase, semi-convection does not develop. During CH	extsc{e}B, however, the growth of the convective core combined with the presence of a minimum in the \( \nabla_{\text{rad}} \) profile inside it can lead to the development of semi-convection (Castellani et al. 1971). Because the free-free transitions become an appreciable source of opacity in a C-O mixture, helium-rich layers just outside the convective core are more stable than C-O rich layers just inside the convective border. Fresh helium engulfed by the growing convective core thus lowers \( \nabla_{\text{rad}} \) (Eq.(1)) until the layers at the minimum of \( \nabla_{\text{rad}} \) become radiatively stable. Thereafter, the core is made of two convective zones. The central one shrinks until helium exhaustion, while the outer one can develop into a SCZ because a strong discontinuity in the chemical abundances profiles exists at its outer boundary. Our evolutionary computations show, however, that semi-convection does not develop in intermediate-mass stars. The layer at the minimum of \( \nabla_{\text{rad}} \) is found to become radiatively stable when the central helium has dropped to less than \( 10^{-4} \) in mass fraction in the 5 M\(_{\odot}\) star, and to about 0.02 in the 2.5 M\(_{\odot}\) one. The development of semi-convection will, however, be more effective in low-mass stars. In those objects, the minimum in \( \nabla_{\text{rad}} \) reaches the value of \( \nabla_{\text{ad}} \) at an earlier stage of central helium combustion, when the whole convective core is still growing (see e.g. Sweigart & Demarque 1972; Sweigart & Gross 1976; Lee & Demarque 1990).

An interesting result is found in our 2.5 M\(_{\odot}\) star. Along the first giant branch, the central temperature is influenced by two opposite effects. The first one is the heating due to core contraction, favoured by the growth of the core mass. The second one is the cooling due to the very high conductivity resulting from the increasing core electron degeneracy. Surprisingly, the central temperature of our 2.5 M\(_{\odot}\) star (with no overshooting) does not reach high enough temperatures to ignite the 3\( \alpha \)-process. After

---

*Fig. 6a–d.* Evolution of the core potential \( \phi_c = M_c/R_c \) in our 15 M\(_{\odot}\) star. The dotted line represents the critical value of \( \phi_c \) above which the star becomes a red giant (defined by \( \log(T_{\text{eff}}) < 3.6 \)) for the first time after core hydrogen exhaustion.

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reaching $85 \times 10^6$ K, it drops to $70 \times 10^6$ K. The star spends about 40 million years ascending the giant branch before the occurrence of the central helium flash, which has been followed in part up to luminosities and central degeneracy $\rho_c$ of 3500 $L_{\odot}$ and 24.4, respectively. The same sequence computed with the Los Alamos opacities (Huebner et al. 1977), the other input physics being unchanged, ignites helium in a moderately degenerate core. We assign this difference to the deeper penetration of the convective envelope due to the new Livermore opacities. Comparisons with earlier calculations with the Los Alamos opacities for intermediate mass stars (Forestini et al. 1992) confirm the expected deepening of the convective envelope. We conclude that 2.5 $M_{\odot}$ should be the lower limit for helium ignition in a degenerate core when no overshooting is applied, instead of 2.2 $M_{\odot}$ (Maeder & Meynet 1989).

3.3. Mass loss

It has long been recognized that mass loss can affect significantly the evolution of massive stars (see for example the review by Chiosi & Maeder 1986). We find that the mass loss rate itself is strongly influenced by the structural evolution of those stars. Figure 7 displays the various mass loss rates experienced by our 10, 15 and 20 $M_{\odot}$ stars from the end of the main sequence. Models including semi-convection spend most of their post-MS lifetime at high effective temperatures, and consequently suffer less mass loss than in the standard cases. Overshooting, on the other hand, increases the rate of mass loss, as the models run at higher luminosities. The influence of semi-convection or overshooting on the mass lost by our model stars can be seen in Table 1.

### Table 1. Mass at end of core hydrogen ($M_{\text{end H}}$) and core helium ($M_{\text{end He}}$) burning, and total mass lost versus the initial stellar mass ($M_{\text{init}}$). All masses are in $M_{\odot}$. The labels m, s, o and c refer to the models calculated with the standard mixing scheme, semi-convection, overshooting applied to all convective zones, and core overshooting, respectively.

<table>
<thead>
<tr>
<th>$M_{\text{init}}$</th>
<th>$M_{\text{end H}}$</th>
<th>$M_{\text{end He}}$</th>
<th>$M_{\text{init}} - M_{\text{end He}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 m</td>
<td>9.87</td>
<td>9.53</td>
<td>0.34</td>
</tr>
<tr>
<td>10 s</td>
<td>9.88</td>
<td>9.59</td>
<td>0.29</td>
</tr>
<tr>
<td>10 o</td>
<td>9.82</td>
<td>9.53</td>
<td>0.29</td>
</tr>
<tr>
<td>10 c</td>
<td>9.81</td>
<td>9.43</td>
<td>0.38</td>
</tr>
<tr>
<td>15 m</td>
<td>14.16</td>
<td>13.78</td>
<td>1.38</td>
</tr>
<tr>
<td>15 s</td>
<td>14.18</td>
<td>13.91</td>
<td>1.27</td>
</tr>
<tr>
<td>15 o</td>
<td>14.11</td>
<td>13.68</td>
<td>1.33</td>
</tr>
<tr>
<td>15 c</td>
<td>14.04</td>
<td>13.62</td>
<td>1.42</td>
</tr>
<tr>
<td>20 m</td>
<td>18.70</td>
<td>18.34</td>
<td>1.36</td>
</tr>
<tr>
<td>20 s</td>
<td>18.71</td>
<td>18.42</td>
<td>1.29</td>
</tr>
<tr>
<td>20 o</td>
<td>18.44</td>
<td>17.86</td>
<td>2.29</td>
</tr>
<tr>
<td>20 c</td>
<td>18.59</td>
<td>17.70</td>
<td>2.39</td>
</tr>
</tbody>
</table>

### Table 2. Estimated increase of the CHeB lifetime (in %) due to breathing pulses.

<table>
<thead>
<tr>
<th>$M_{\text{init}}$ ($M_{\odot}$)</th>
<th>2.5</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard case</td>
<td>3</td>
<td>4</td>
<td>8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>With semi-convection</td>
<td>10</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>With overshooting</td>
<td>-</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>With core overshooting</td>
<td>2</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

3.4. Breathing pulses

So-called "breathing pulses" have been found by several authors during central helium burning (see Castellani et al. 1985). This phenomenon introduces large uncertainties in our understanding of this phase, especially concerning its total lifetime and central abundances. The existence of these breathing pulses is still a matter of debate. Our computations do not help solving that question (see Table 2). It seems, however, that intermediate-mass stars are more subject to breathing pulses than massive stars. The estimated increase of the CHeB lifetime lies between 2 and 10%.
4. Comparison with observations

4.1. General considerations

The observation of stars in different regions of the HRD offers a useful tool to test the validity of computed stellar models (see the review by Chiosi et al. 1992). The question is most relevant for massive stars, for which slight changes in the input physics and in the numerical treatment of unstable zones can lead to completely different evolutionary features. Two important questions are (1) whether the star is a red or a blue supergiant (RSG or BSG) at central helium ignition, and (2) whether a blue loop occurs in the HRD during CHeB. We have seen in Sect. 4.2 the important role played by the energetics of the H-burning shell. A discontinuous chemical profile just above this H-burning shell enhances the energy production, and prevents the star from moving to the red at He ignition. Such is the case in our standard models (see Sect. 4.2). Chin & Stothers (1990) further note that, during the early He burning, the star is bluer for a larger initial He abundance, or a smaller metallicity, and that large mass loss disfavours the development of FCZs. Tuchman & Wheeler (1989) confirm these results from the analysis of envelope models in thermodynamic equilibrium. They further notice that moderate mass loss rates lower the final effective temperature at which, during CHeB, the star jumps to the Hayashi line on a thermal time scale. They also emphasize that very high mass loss rates enable the star to make an immediate thermal transition to the red Giant before helium ignition. A blue loop, on the other hand, can be triggered during CHeB depending on the chemical profile of the inhomogeneous region into which the H-burning shell enters. A deeper penetration of the convective envelope, for example, favours the development of a blue loop (Sect. 3.2).

Our results are summarized in Table 3. Basically, most of our massive stars ignite helium while still being a BSG. In the course of their CHeB phase, however, they reach an effective temperature from which they move to the Hayashi line on a thermal time scale, and pass the rest of their life as RSG. Exception to this pattern are found (1) in the 15 M\(_\odot\) star with core overshooting. This model spends its CHeB phase as a RSG. (2) in models with semi-convection which move to a RSG before igniting helium and perform a BL to high effective temperature during CHeB. As shown below, the confrontation of these different evolutionary features with observations enables to discriminate between the various mixing schemes.

4.2. Lifetimes

The lifetimes of different evolutionary phases are given in Table 4 for the different mixing schemes. As we have seen, semi-convection has a negligible effect on the time spent during core hydrogen burning (\(t_H\)). This is no more the case for the time spent during core helium burning (\(t_{He}\)) which increases by \(18\%\) for the 5 M\(_\odot\) and decreases by (3.5, 11.5, 13.5)\% for the (10, 15, 20) M\(_\odot\) star. We notice the sensitivity of \(t_{He}\) on the location of the core boundary. Although semi-convection does not develop during CHeB in intermediate-mass stars, the application of the semi-convection algorithm results in larger \(t_{He}\) (see Sect. 2.3.2). Overshooting, on the other hand, results in a decrease of \(t_{He}\) by (22, 32, 28, 19)\% for our (5, 10, 15, 20) M\(_\odot\) stars.

The observed relative number of WR to OB stars offers a good opportunity to check the different mixing scenarios. Conti et al. (1983) propose a ratio lying between 0.23 and 0.36 for M > 40 M\(_\odot\). The values of \(t_{He}/t_H\) found in our models are given in Table 4. It decreases from intermediate-mass to massive stars, reaching a lower value between 0.11 (with overshooting) and 0.14 (in the standard case). According to recent computations (Maeder 1987; Schaller et al. 1991), this ratio decreases for 2.5 to 10 M\(_\odot\) stars, reaching a lower value which remains about constant up to 50 M\(_\odot\) stars. It then increases again for more massive stars. On such grounds, we can thus extrapolate our results to more massive stars, and compare them to the observed number of WR relative to OB stars. Our predictions in the standard cases and in models with semi-convection are slightly lower than the observed ratios. Models including semi-convection do not show the expected plateau in \(t_{He}/t_H\) above 15 M\(_\odot\), and models at higher masses should be considered before drawing a conclusion. The application of overshooting decreases \(t_{He}/t_H\) still further, increasing the discrepancy between prediction and observations. Overshooting in massive stars seems thus disfavoured.

4.3. Morphology in the H-R diagram

The relative times \(t_{He,blue}\) and \(t_{He,red}\) spent by massive stars as BSG and RSG offer another very sensitive test of the different mixing schemes presented in this paper. The ratios reported in Table 4 can be affected in massive stars (1) by the location in the HRD of the star when helium ignites at the centre, and (2) by the occurrence of blue loops during central helium burning. We have seen (Sect. 3.2) how sensitive these features are to the conditions prevailing in the interior of these stars during the Post-Main-Sequence phase. To have a better idea of the morphology expected with the different mixing schemes, we display in Fig. 8 composite HRDs in which the density of points on each track is proportional to the time spent by the models in the corresponding phase (for each track, 1000 points are plotted, spaced by equal time intervals). This figure can be used efficiently in comparison with observational HRDs.

First, observations of massive stars in our Galaxy show the existence of many giants located in the HRD between the MS band and the red giants (Humphreys & Mc Elroy 1984; Blaha & Humphreys 1990). Figure 8 shows that almost all the mixing schemes predict such an existence of CHeB stars in that region of the HRD. Exception is however found for the 15 M\(_\odot\) models with core overshooting, which never become blue supergiants after central hydrogen exhaustion. Models with core overshooting lead thus to unobserved effects.

Second, observational HRDs reveal the existence of a gap to the left of the Hayashi track for massive stars, where very few stars are found. In the data collected by Humphreys & Mc
Table 3. Post-Main-Sequence characteristics of the 15 and 20 $M_\odot$ stars.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>during</th>
<th>Envelope</th>
<th>begin</th>
<th>during</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Post-MS</td>
<td>penetration?</td>
<td>CHeB</td>
<td>CHeB</td>
</tr>
<tr>
<td>Standard</td>
<td>large IFCZ</td>
<td>No</td>
<td>BSG</td>
<td>No BL</td>
</tr>
<tr>
<td>With semi-convection</td>
<td>large ISCZ</td>
<td>Yes</td>
<td>RSG</td>
<td>IFCZ-BL</td>
</tr>
<tr>
<td>With overshooting</td>
<td>large IFCZ</td>
<td>No</td>
<td>BSG</td>
<td>No BL</td>
</tr>
<tr>
<td>With core overshooting *</td>
<td>small IFCZ</td>
<td>Yes</td>
<td>RSG</td>
<td>No BL</td>
</tr>
</tbody>
</table>

* The Post-Main-Sequence characteristics of the 20 $M_\odot$ models are similar to the ones displayed in the standard case.

Table 4. Lifetimes of different evolutionary phases (expressed in $10^6$ yr). $t_H$ and $t_{He}$ indicate the durations of core H and He burning, respectively. $t_{He, red}$ and $t_{He, blue 1}$ represent the times spent during central He-burning as red and blue supergiants, respectively, while $t_{He, blue 2}$ and $t_{He, blue 3}$ are the times spent as central He-burning blue supergiants before and after the first dredge-up phase, respectively. The labels m, s, o and c have the same meaning as in Table 1.

<table>
<thead>
<tr>
<th>$M_{init}$</th>
<th>$t_H$</th>
<th>$t_{He, blue 1}$</th>
<th>$t_{He, blue 2}$</th>
<th>$t_{He, red}$</th>
<th>$t_{He}$</th>
<th>$t_{He}/t_H$</th>
<th>$t_{He, blue 1}/t_{He}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 m</td>
<td>513</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>s</td>
<td>521</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>o</td>
<td>628</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>607</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5 m</td>
<td>86.2</td>
<td>0</td>
<td>3.9</td>
<td>17.7</td>
<td>21.6</td>
<td>0.25</td>
<td>0.18</td>
</tr>
<tr>
<td>s</td>
<td>86.2</td>
<td>0</td>
<td>10.2</td>
<td>15.3</td>
<td>25.5</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>o</td>
<td>101.6</td>
<td>0</td>
<td>7.2</td>
<td>10.9</td>
<td>18.1</td>
<td>0.18</td>
<td>0.40</td>
</tr>
<tr>
<td>c</td>
<td>101.7</td>
<td>0</td>
<td>4.9</td>
<td>12.4</td>
<td>17.3</td>
<td>0.17</td>
<td>0.28</td>
</tr>
<tr>
<td>10 m</td>
<td>20.71</td>
<td>0</td>
<td>1.56</td>
<td>2.19</td>
<td>3.75</td>
<td>0.18</td>
<td>0.42</td>
</tr>
<tr>
<td>s</td>
<td>20.72</td>
<td>0</td>
<td>1.89</td>
<td>1.73</td>
<td>3.62</td>
<td>0.17</td>
<td>0.52</td>
</tr>
<tr>
<td>o</td>
<td>23.67</td>
<td>0</td>
<td>1.26</td>
<td>1.42</td>
<td>2.68</td>
<td>0.11</td>
<td>0.47</td>
</tr>
<tr>
<td>c</td>
<td>23.68</td>
<td>0</td>
<td>0.99</td>
<td>1.73</td>
<td>2.72</td>
<td>0.11</td>
<td>0.36</td>
</tr>
<tr>
<td>15 m</td>
<td>11.65</td>
<td>0.69</td>
<td>0</td>
<td>1.22</td>
<td>1.91</td>
<td>0.16</td>
<td>0.36</td>
</tr>
<tr>
<td>s</td>
<td>11.45</td>
<td>0</td>
<td>1.08</td>
<td>0.61</td>
<td>1.69</td>
<td>0.15</td>
<td>0.64</td>
</tr>
<tr>
<td>o</td>
<td>12.74</td>
<td>0.63</td>
<td>0</td>
<td>0.92</td>
<td>1.55</td>
<td>0.12</td>
<td>0.41</td>
</tr>
<tr>
<td>c</td>
<td>12.75</td>
<td>0</td>
<td>0</td>
<td>1.45</td>
<td>1.45</td>
<td>0.11</td>
<td>0</td>
</tr>
<tr>
<td>20 m</td>
<td>8.14</td>
<td>0.84</td>
<td>0</td>
<td>0.28</td>
<td>1.12</td>
<td>0.14</td>
<td>0.76</td>
</tr>
<tr>
<td>s</td>
<td>8.18</td>
<td>0</td>
<td>0.52</td>
<td>0.45</td>
<td>0.97</td>
<td>0.12</td>
<td>0.54</td>
</tr>
<tr>
<td>o</td>
<td>9.28</td>
<td>0.36</td>
<td>0</td>
<td>0.63</td>
<td>0.99</td>
<td>0.11</td>
<td>0.37</td>
</tr>
<tr>
<td>c</td>
<td>8.91</td>
<td>0.40</td>
<td>0</td>
<td>0.57</td>
<td>0.97</td>
<td>0.11</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Elroy (1984), this gap extends to log($T_{eff}$) = 3.8 for galactic supergiants. In these ranges of luminosity and effective temperatures, the star cannot maintain thermodynamic equilibrium in the envelope. The star moves, on a thermal time scale, either to the Hayashi line (see Tuchman & Wheeler 1989), or to higher effective temperatures, performing a blue loop. Our models with overshooting applied to the boundaries of all convective zones, however, place the blue side of the gap at log($T_{eff}$) = 3.7, which is too low compared to the observations. Overshooting at the boundaries of all convective regions are thus also unable to account for the observations.

Third, no gap is found in the observed HRDs at higher effective temperatures. This feature is hard to be reproduced from the evolutionary models, which predict that a star moves out of the MS band on a thermal timescale. Our standard case models can only explain the existence of BSGs up to log($T_{eff}$) = 4.10. Our models including semi-convection, however, predict the existence of BSG up to effective temperatures as high as log($T_{eff}$) = 4.25. There is still a region in the right side of the MS in which neither of our models, nor any one of single star published in the literature, predict the existence of BSG stars. We refer to Tuchman & Wheeler (1990) for a discussion of this problem. They suggest that some of these stars may be binary systems which went through a mass accretion process leading to the formation of an outer helium-rich layer.

It is tempting to further compare the result of our simulations with the observational data obtained by Fitzpatrick & Garmany (1990) on luminous stars in the Large Magellanic Cloud. Apart from strengthening the general features observed by Humphreys & Mc Elroy (1984), their analysis reveals the existence in the
HRD of a ledge in the BSG region, in the hotter side of which the density of stars is over 5 times greater than on the cooler side. The location of this ledge and of the blue side of the gap on the left to the Hayashi line extends from \((\log(T_{eff}), \log(L)) = (3.84, 4.10)\) to \((4.11, 5.50)\) and from \((3.84, 4.10)\) to \((3.70, 5.50)\) respectively. It is seen that the predictions of our models including semi-convection are in much better agreement with the observations than the ones of our models with the other mixing schemes. However, we should be cautious that the stars in the Magellanic Clouds have lower metallicities than our Pop. I model stars. According to Schaller et al. (1992), a decrease in metallicity enables massive stars with core overshooting to pass a non-negligible fraction of their CHeB as BSG prior to become RSG. The evolutionary features of models with core overshooting but at lower metallicity would thus be similar to our Pop. I standard case simulations. The sensitivity of our models to metallicity should thus be checked before drawing a firm conclusion. Such a study is currently under way.

4.4. Surface abundances

The analysis of the surface abundances of the yellow to blue super giants should provide a further test for discriminating between the different evolutionary scenarios. According to our models, only the ones including semi-convection show alteration in their surface abundances. Unfortunately, such an analysis has only been reported for a few RSG. The comparison of the observed surface abundances of those stars with the predictions of our models will be presented elsewhere (Mowlavi et al. 1993).

Let us, however, mention an interesting feature. The observations of Fransson et al. (1989) show evidence for CNO processing in the progenitor of SN 1987A. This suggests that the star has evolved from red to blue. This can only be reproduced by models performing a BL during the CHeB. This feature could be accounted for by the models with semi-convection. Again, the effect of a lower metallicity on the predictions of our models should be checked.

5. Conclusions

Our new generalized treatment of semi-convection has been applied to the evolution of intermediate-mass and massive Pop. I stars until the end of central helium burning. These models have been compared with evolutionary sequences including overshooting applied to the border of all convective zones, core overshooting, or just the standard mixing scheme. It appears that the structural evolution of the stars, especially the massive ones, is strongly affected by the type of mixing assumed. In particular, it appears that, for Pop. I stars:

1. During the main sequence phase, semi-convection becomes important in stars more massive than 10 M\(_\odot\). Its development, however, does not change the MS characteristics. If core overshooting is applied during that phase, the semi-convective zones do not develop in stars up to 15 M\(_\odot\). For larger masses, however, semi-convection might develop after central hydrogen exhaustion even in the presence of core overshooting.

2. After central hydrogen exhaustion, the four mixing schemes lead to completely different evolutionary features for massive stars. In the standard case and when overshooting is applied to the boundaries of all convective zones, large IFCZ develop and the star ignites its central helium while still being a BSG. The CHeB is characterized by a continuous penetration of the convective envelope. Models including semi-convection reveal the development of large semi-convective zones. They ignite He as RSG, after which the deep convective envelope recedes. A large IFCZ further develops during CHeB, and the star follows a large loop in the HRD. When core overshooting
is included, a small IFCZ develops during the core contraction phase. The star spends all its post-MS life as RSG. The CHEB phase is characterized by a massive convective envelope. The mass of the IFCZ in our 20 $M_{\odot}$ star is however larger than in our 15 $M_{\odot}$ and the star shows evolutionary characteristics similar to the ones in the standard case.

(3) Semi-convection does not develop during CHEB in stars more massive as 2.5 $M_{\odot}$.

(4) Models of massive stars including semi-convection suffer less mass loss than the standard ones, as they spend most of their life as blue supergiants. Core overshooting, on the other hand, increases the rate of mass loss as the models run at higher luminosities.

The confrontation with observations, especially for massive stars, lead to the following conclusions:

(1) Models computed with the standard mixing scheme or with overshooting applied to the boundaries of all convective zones associate blue supergiants with massive stars igniting helium in their core. They have effective temperatures that are too low to explain all the observed non-MS blue supergiants.

(2) Models including core overshooting fail to predict the existence of blue supergiants on the right of the MS, supposed to be in their central helium burning phase. Besides, they lead to too low $L_{H}/L$ ratios to explain the observed number ratio of WR to OB stars.

(3) Models of massive stars which perform a blue loop during CHEB explain best the observed distribution of luminous stars in the HRD. Such a feature is observed only in our models including semi-convection.

Note in proof: After having submitted this paper, an article was published by Alongi et al. (1993, AAS 97, 851) in which the issue of convective overshoot is discussed. They present evolutionary models of stars with Z=0.008 including overshooting at the boundaries of both the convective core and envelope. In order to reproduce the observational features in the HRD, they have to introduce an efficiency of convective overshoot in the envelope different from the one in the core. These models are compared with those including semi-convection during CHEB. They confirm that the semi-convective regions during CHEB are unimportant in stars more massive than 5 $M_{\odot}$. They did not, however, include semi-convection in stars more massive than 10 $M_{\odot}$.

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