Application of an extended mixing length model to the convective envelope of the Sun and its Li and Be content

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Abstract. New solar envelope models are calculated with an improved treatment of convection. This leads to revised constraints on the depth of the (convective + underlying transport) region having to connect the surface with the region where Li and Be are destroyed in quantities required by the observations.

Our convection model, inspired by the mixing length theory, drops the assumption of incompressibility by means of the introduction of two additional free parameters.

In a first step, taking due account of dilution effects in the envelope, and assuming that no Li and Be are destroyed in the pre-main sequence phase, we deduce that 6Li and 9Be have to be mixed from the surface down to regions where the temperatures lie in the 2.6 \times 10^6 \leq T \leq 3.0 \times 10^6 \text{ K} and 3.6 \times 10^6 \leq T \leq 3.7 \times 10^6 \text{ K} ranges, respectively, the precise values of T depending on the time scale(s) of the transport mechanism(s) below the convective zone. The temperatures generally reported in the literature are lower than these (2.5 \times 10^6 \text{ K} and 3.5 \times 10^6 \text{ K}, respectively). We thus conclude that the transport region called for in order to explain the Li and Be observations is larger in mass than predicted up to now, except of course if the convective envelope is in its turn deeper than expected from the standard model.

In a second step, we show that the compression effects we simulate in our extended convection model may well be able to lead to a convective deepening without having to resort to any type of overshooting. Of course, this possibility has to be evaluated further by means of detailed calculations of the global evolution of the Sun.

Key words: stars: convection – Sun: convection – Sun: abundances

1. Introduction

The Li surface abundance of the Sun is about 100 times lower than the maximum value derived from the observations of F and G field dwarf stars with effective temperatures $T_{\text{eff}} > 6500 \text{ K}$ (Steinbock & Holweger 1984), as well as from the analysis of primitive meteorites (carbonaceous chondrites; Grevesse 1984). The Li content of other F and G dwarf stars of about the same age is also lowered by similar amounts. On the other hand, the solar Be abundance is only two times lower than the carbonaceous chondrite value (Chmielewski et al. 1975). Those solar Li and Be abundance data provide a very interesting opportunity to probe the internal structure of the Sun, namely if they are put in perspective with the fact that Li is slightly more fragile to proton captures than Be in conditions appropriate to the solar envelope. In fact, those observations not only put strong constraints on the location of the base of the convective envelope, but also call for the operation of some (non-convective) “transport” between that bottom layer and the deeper regions of nuclear destruction.

A lot of papers have been devoted to the problem of the Li and Be stellar and solar abundances (e.g. Schatzman 1984; Michaud et al. 1984; D’Antona & Mazzitelli 1984; see e.g. Arnould & Forestini 1989, for a review). Some of them have tried to explain the solar Li and Be abundances in connection with a remarkable correlation between $T_{\text{eff}}$ and the Li (or Be) abundances observed in some open clusters. It has been put forward that this correlation (and especially the gap in clusters like the Hyades) might be the signature of the combined operation of diffusion and convective transport, and mass loss (Michaud 1986). In addition, the dependence upon the cluster age makes slow processes like turbulent diffusion (Schatzman 1984; Michaud et al. 1984) and meridional circulation (Charbonneau & Michaud 1989) the best candidates for the transport mechanism.

In the studies concerned with turbulent diffusion, the diffusion coefficient remains a free parameter, in spite of some recent efforts to build up a self-consistent theory of turbulence (e.g. Zahn 1983). Consequently, the adopted values for that coefficient depend in general on the assumed extent of the transport region, defined here as the region below the convective zone, where the Li abundance is not negligibly small. Up to now, the selected diffusion coefficients are generally based on underestimated limits for that region. In order to substantiate that claim, let us remind first that the depletion factor of 100 for the solar Li can be obtained if the lifetime of $^6$Li against proton captures is $\approx 4.6$ times shorter than the age of the Sun. Such a nuclear lifetime requires a temperature of $\approx 2.5 \times 10^6 \text{ K}$ (see e.g. Caughlan & Fowler 1988), which is

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1 In stellar conditions, $^6$Li is nuclear much more fragile than $^7$Li, and is consequently predicted to be almost absent from the solar photosphere, independently of the details of the solar envelope models. However, such a $^6$Li depletion cannot explain the solar Li underabundance for any reasonable initial Li isotopic composition.
envisioned in the models mentioned above as the temperature at the base of the transport region. This assumption thus neglects any dilution of Li in that region, which can be valid only if the typical timescales for the transport of material there were much longer than the solar lifetime. Of course, in such a limit, the transport region would be of no relevance in the solar Li problem. In fact, D'Antona & Mazzitelli (1984) have already shown that the present surface Li depletion would be significantly lower than observed if Li is properly homogenized over the whole convective envelope and distributed instantaneously in a transport zone, the base of which has the temperature of 2.5 \times 10^6 K mentioned above. As a consequence, they consider in their models a transport which has to extend down to higher temperatures, and which could even be at work during the Pre-Main Sequence phase. The very nature of that mechanism (called extra-mixing by D'Antona & Mazzitelli (1984), which is in fact a mixing zone), is not specified, however.

In order to better characterize the solar transport region and the physical processes at work there, a reliable description of the properties and extent of the solar convective envelope is obviously required. The aim of this paper is to report on some solar envelope models calculated with an improved version of the mixing length theory (MLT), and to stress the possible impact of this model on the predicted envelope properties. The model is briefly described in Sect. 2. Section 3 discusses the extent of the transport region which is required in order to account for the solar data, and then presents our results. Some conclusions are drawn in Sect. 4.

2. Physics of our model

2.1. The envelope code

Solar envelopes and atmospheres are calculated with the envelope code described in detail elsewhere (Forestellini & Arnould 1990). Let us just remind here the following features:

1. The atmospheric temperature profile is computed from

\[ T^{*} = \frac{1}{2} \mathcal{R}^{*} \left[ \tau' + q(\tau'; \mathcal{R}^{*}) \right] , \]

where \( \tau' \) is the optical depth corrected for the sphericity of the atmosphere, and

\[ \mathcal{R}^{*} = L/4\pi\sigma\mathcal{R}^{2} , \]

where \( \mathcal{R} \) is the radius at \( \tau' = 0 \) \( (\tau' = 10^{-6} \text{ in practice}) \), and \( \sigma \) the Boltzmann constant. We have simplified this profile to the Eddington limit, suitable for the solar case, by setting \( q = 2/3 \).

2. Our equation of state includes the contributions from the ionization stages of H, He, C, N, and O, as well as the electrostatic corrections.

3. Radiative opacities are from Alexander (1975) for temperatures below \( 10^6 \text{ K} \), and from Huebner et al. (1977) at higher temperatures.

2.2. The treatment of convection

In the last ten years, real progress has been made in the numerical integration of the completely compressible non-linear equations describing convection (e.g. Graham 1979; Toomre et al. 1984; Chan & Sofia 1986; Huilburgt et al. 1986). This has mainly led to the demonstration that the detailed structure of the convective cells is determined to a large extent by the stratification \( \chi \), defined as the ratio of the density at the bottom of the convective envelope to that at its surface. More specifically, an increase of \( \chi \) leads to a spreading of the ascending part of a globule, while its descending part gets narrow and piercing. As a result, the convection transport becomes highly asymmetric. This also modifies the energy budget through a resultant kinetic energy flux.

Such features are completely neglected in the usual MLT adopted in the modeling of the stellar structure. In particular, the MLT can be derived from the Boussinesq equations for incompressible fluids. However, stellar convection zones, like the solar one, are usually highly stratified \( (\chi > 1000) \), and the compression effects are expected to play a significant role.

In view of the serious shortcomings of the classical MLT mentioned above, we have developed a new formalism based on the MLT, but in which two of its main restrictions have been suppressed (Forestini et al. 1990). The first one concerns the assumption that there is no velocity distribution inside a convective cell, which translates in the MLT into

\[ \bar{v}^{2} = \bar{v}^{2} , \]

where \( \bar{v} \) represents the mean velocity of a convective cell. We have restored such a spatial distribution (supposed to be the same for the downward and upward motions) by introducing

\[ \bar{v}^{2} = \bar{v}^{2} , \]

and

\[ \bar{v}^{2} = \bar{v}^{2} , \]

where the \( \uparrow \) and \( \downarrow \) arrows correspond to the upward and downward velocities, respectively. In Eq. (2), \( a_{0} \) is the ratio of the second moment of the velocity distribution to the first one. In the MLT, \( a_{0} = 1 \), while \( a_{0} > 1 \) values translate the existence of a velocity distribution.

Our second improvement aims at simulating the asymmetry caused by the compression effects. This is achieved by introducing the mixing lengths \( \lambda_{i} = \alpha_{i} H_{p} \) and \( \lambda_{i} = \alpha_{i} H_{p} \) for the downward and upward displacements, respectively, \( H_{p} \) being the pressure scale height. We further impose that the convective motions conserve the mass of the convective envelope, so that the total mass flux must vanish across any surface \( S(r) \) with unit vector \( \hat{n} \):

\[ \oint \vec{q} \cdot \hat{n} d\Omega = 0 , \]

or

\[ \oint \vec{q} \cdot \hat{n} d\Omega = - \oint \vec{q} \cdot \hat{n} d\Omega . \]

In such conditions,

\[ p = \bar{v}^{2} / \bar{v}^{2} , \]

is constrained by the mass conservation.

We have derived a 7-order polynomial equation in \( p \) having only one real solution in the \( 0 < p < 1 \) range when \( \alpha_{i} > \alpha_{j} \), which corresponds to a situation where the descending part of a cell is accelerated (and the ascending part decelerated) by the action of the compression effects. Of course, \( p = 1 \) is the unique solution when \( \alpha_{i} = \alpha_{j} \), as in the classical MLT. Consequently, the energy budget in our extended model reads

\[ F_{\text{RAD}}(r) + F_{\text{CONV}}(r) + F_{K}(r) = L/4\pi r^{2} , \]
with
\[ F_{\text{RAD}}(r) = \frac{16}{3} \frac{\sigma T^4 GM_r}{\kappa r^2} V_{\text{CONV}}, \] (4.1)
\[ F_{\text{CONV}}(r) = \frac{1}{2} q \frac{\tilde{V}}{ \kappa r \alpha_t (V_{\text{CONV}} - V_i)} \mathcal{N}, \] (4.2)
and
\[ F_k(r) = \frac{1}{2} q \frac{\tilde{V}^3}{ \kappa r \alpha_t} (1 - p), \] (4.3)
where
\[ \mathcal{N} = \frac{1}{1 - p} \left( 1 + \frac{\alpha_t}{\alpha_t} \right). \] (4.4)

\[ q \] is the density, \( C_P \) the heat capacity, \( \kappa \) the radiative opacity, \( V_{\text{CONV}} \) the convective gradient, and \( V_i \) the gradient inside the downward part of a convective cell (see Forestini et al. 1990), the other symbols having their usual meaning. The standard MLT relations can be recovered from Eqs. (4) by setting \( p = 1 \), so that \( \alpha_{\text{STD}} = \alpha_t = \alpha_t \) and \( \alpha_{\text{STD}} = \alpha_t = \alpha_t \).

Our model is able to provide downward and upward velocity profiles similar to those obtained in 2-D and 3-D simulations if the parameters \( \alpha_0, \alpha_t, \) and \( \alpha_t \) are chosen adequately (Forestini et al. 1990). Taking \( \alpha_t = \alpha_{\text{STD}} \) (the standard MLT value) and \( \alpha_t = \alpha_t \), we simulate quite well the main characteristics of the effects of the work done by the compression effects, even in the case of an extremely peaked velocity distribution (i.e. \( \alpha_0 \approx 1 \)). So, we can obtain a deepening of the convective envelopes in a natural way. We are forced to treat this extension region (located between the bottom of the convection zone as predicted by our model and the one derived from the MLT) as being superadiabatic \( (V_{\text{CONV}} > V_{\text{RAD}}) \), while it is subadiabatic in reality. However, the error in the calculated temperature profile introduced by this approximation is negligible in the solar case (see a detailed discussion of this point in Forestini et al. 1990).

3. Results

3.1. Timescales and critical temperatures for the destruction of \(^7\)Li and \(^9\)Be

The transport in stellar interiors of nuclides that can experience a nuclear burning is characterized by two different timescales. They can be extracted from the general diffusion equation
\[ \frac{\partial X}{\partial t} = \frac{1}{\rho r^2} \frac{\partial}{\partial r} \left( r^2 \rho \frac{\partial X}{\partial r} \right) - X_{\text{a}}^{\text{nuc}}, \] (5)
where \( \rho \) is the diffusion coefficient, which depends on the exact nature of the transport, and \( X_{\text{a}}^{\text{nuc}} \) is the mass fraction of nuclide \( a \) with mean nuclear lifetime \( \tau_a \), which will be referred to in the following as the first characteristic timescale \( t_1 \). The second timescale is
\[ t_2 \approx r^2/\rho. \] (6)

In a convective zone, \( t_2 \) is given locally by \( \Lambda/\theta \) in the classical MLT, and is always shorter than \( t_1 \) by many orders of magnitude. In such a situation, the convective zone can be seen to a very good approximation as perfectly homogeneous in its composition. The first term of Eq. (5) can thus be neglected, and a mean nuclear lifetime \( \tau_a \) can be defined by
\[ 1/\tau_a = \frac{1}{M_{\text{CE}}} \int_{R_b}^{R} q \frac{\bar{v}}{\kappa r} \, dV, \] (7)
where
\[ M_{\text{CE}} = \int_{R_b}^{R} q \, dV \]
represents the mass of the convective envelope with inner and outer boundaries \( R_b \) and \( R \), respectively. In particular,
\[ 1/\tau_{\text{Li}} = \frac{1}{M_{\text{CE}}} \int_{R_b}^{R} q^2 X_P \langle \sigma v \rangle [^7\text{Li}(p, x) x]^2 \, dr, \] (9.1)
and
\[ 1/\tau_{\text{Be}} = \frac{1}{M_{\text{CE}}} \int_{R_b}^{R} q^2 X_P \langle \sigma v \rangle [^9\text{Be}(p, d) 2x] + \langle \sigma v \rangle [^9\text{Be}(p, x) ^6\text{Li}] \], (9.2)
with \( \langle \sigma v \rangle[A, x, y]B \) represents the Maxwellian-averaged rate of the nuclear reaction \( A(x, y) B \) in usual notations, and \( X_P \) is the mass fraction of hydrogen in the convective zone.

Let us assume that \( t_1 \gg t_2 \) remains true in the transport region below the convective envelope. In such conditions, the integrals in Eqs. (8), (9.1), and (9.2) can be extended down to the radius \( R_b \) defining the bottom of the transport region. In order to evaluate Eqs. (9.1) and (9.2), we suppose in addition that the \(^7\text{Li} \) depletion occurs only during the main sequence phase of the Sun (duration \( t_\odot = 4.6 \times 10^9 \) yr), and we calculate solar envelopes using the standard MLT with \( \alpha_{\text{STD}} = 1.5^4 \). The physical conditions at the base of the convective envelope derived from such a model are displayed in Table 1. In order to satisfy the constraint
\[ \exp(-t_\odot/\tau_{\text{Li}}) = 0.01 \]
set by the Li observations, we find that the transport must reach regions where \( T_b^* (^7\text{Li}) \approx 3.0 \times 10^6 \) K (see Sect. 3.2). Similarly, a \(^9\text{Be} \) destruction by a factor of two is obtained if \( T_b^* (^9\text{Be}) \approx 3.7 \times 10^6 \) K. Disregarding the possibly remaining uncertainties in the rate of proton captures by \(^9\text{Be} \) in solar envelope conditions (Rolf’s, private communication), those temperatures are well defined because of the extremely steep dependence of the \(^7\text{Li} \) and \(^9\text{Be} \) proton capture rates on temperature in the relevant conditions. They have to be compared with the values \( T^*_\odot (^7\text{Li}) = 2.5 \times 10^6 \) K and \( T^*_\odot (^9\text{Be}) = 3.5 \times 10^6 \) K generally reported in the literature, and which are chosen to determine the free parameters in the diffusion equation relevant to the transport region, as stressed in Sect. 1.4.

Of course, the assumption that \( t_1 \gg t_2 \) in the transport region is probably wrong. In fact, \( t_1 \approx t_2 \) is likely to be more realistic there. This is mainly supported by the observations of the progressive Li (and, in some cases, Be) depletion in open cluster stars of different ages, as reported in Sect. 1. In this case, the complete diffusion Eq. (5) must be kept, leading to \( 2.5 \times 10^6 < T_b^* (^7\text{Li}) < 3.0 \times 10^6 \) K in order to satisfy Eq. (10). Consequently, the values of \( 3.0 \times 10^6 \) K and \( 3.7 \times 10^6 \) K reported above for \( T_b^* (^7\text{Li}) \) and \( T_b^* (^9\text{Be}) \) have to be seen as upper limits.

\footnote{3 This value for \( \alpha_{\text{STD}} \) seems to be slightly smaller than those actually used in recent standard models (e.g. Turck-Chièze et al. 1988). However, this does not really affect our discussion, which mainly deals with the location (generally around 0.69 to 0.73 \( R_\odot \)) and temperature (generally around 2 \( 10^6 \) K) of the base of the convective envelope.

\footnote{4 However, some more self-consistent studies (e.g. Schatzman 1984) do not use \( T_b^* \) any more in order to determine the value of the diffusion coefficient.}
Table 1. Compression effects on the structure of the solar convective envelope. Model STD refers to the adoption of the standard MLT for convection, with \( \alpha_{\text{STD}} = 1.5 \). The other results are obtained with our convection model (Sect. 2) for several values of \( \alpha_1 \), \( \alpha_2 = 1.5 \) and \( a_0 = 1 \) being adopted in each case. The bottom of the convective envelope is characterized by its radius \( R_b \) (in solar units), temperature \( T_b \) (in units of \( 10^6 \) K), and density \( \varrho_b \) (in cgs). The following three columns give the corresponding values (in the same units) at the bottom of the transport region, as required by Eq. (10) in the \( \tau_1 \gg \tau_2 \) regime (see Sect. 3.2). The extension of the transport region which has to be accounted for by another mechanism (e.g. meridional circulation or turbulent diffusion), expressed in terms of the pressure scale height of the deepest convective shell, is given in the last column. In all cases, a helium mass fraction \( Y = 0.265 \) is selected (this is close to the different ranges of \( Y \) values proposed by the standard models).

<table>
<thead>
<tr>
<th>Model</th>
<th>( R_b )</th>
<th>( T_b )</th>
<th>( \varrho_b )</th>
<th>( R_b^* )</th>
<th>( T_b^* )</th>
<th>( \varrho_b^* )</th>
<th>( \Delta \varrho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD</td>
<td>0.6952</td>
<td>1.997</td>
<td>0.1324</td>
<td>0.5883</td>
<td>2.773</td>
<td>0.3211</td>
<td>1.58</td>
</tr>
<tr>
<td>( \alpha_1 = 1.80 )</td>
<td>0.6579</td>
<td>2.323</td>
<td>0.1742</td>
<td>0.5817</td>
<td>2.892</td>
<td>0.3664</td>
<td>1.12</td>
</tr>
<tr>
<td>( \alpha_1 = 2.00 )</td>
<td>0.6321</td>
<td>2.552</td>
<td>0.2236</td>
<td>0.5739</td>
<td>3.009</td>
<td>0.3973</td>
<td>0.87</td>
</tr>
<tr>
<td>( \alpha_1 = 2.25 )</td>
<td>0.6127</td>
<td>2.753</td>
<td>0.3282</td>
<td>0.5815</td>
<td>3.006</td>
<td>0.4274</td>
<td>0.47</td>
</tr>
<tr>
<td>( \alpha_1 = 2.63 )</td>
<td>0.5891</td>
<td>2.996</td>
<td>0.4536</td>
<td>0.5886</td>
<td>3.003</td>
<td>0.4602</td>
<td>( \sim 0.0 )</td>
</tr>
</tbody>
</table>

3.2. Mixing and compressive convection

The main consequence of our new evaluation of \( T_b^* \) is that the transport process must operate deeper than considered before, especially in studies of turbulent diffusion. As a result, that process, which is also constrained by other considerations (like \(^3\)He abundance, seismology, \ldots), may well be unable to operate deep enough to explain the required \(^7\)Li depletion. Of course, it is quite difficult to draw firm conclusions in that respect right now, as the basic turbulent diffusion mechanisms remain very poorly known.

Even if turbulent diffusion cannot be disregarded, we propose an alternative remedy to the problem raised by the increased \( T_b^* \) values mentioned above. It relies on the convection model described in Sect. 2. Some of the results displayed in Table 1 are obtained in such a framework for several choices of \( \alpha_1 \) in the 1.8–2.6 range, \( a_0 = 1 \) and \( \alpha_2 = 1.5 \) being adopted in each case. The comparison of those results with the ones obtained in the standard MLT, and in particular the examination of the last column of Table 1, clearly demonstrates that the consideration of the compression effects in the convection model leads to a deepening of the solar convective zone, and thus helps reducing the size of the transport region required in order to satisfy Eq. (10). The case \( \alpha_1 = 2.63 \) even predicts a convective envelope extending all the way down to \( T_b = T_b^* \) (\(^7\)Li). This situation is clearly unrealistic, as the observations call for a transport region in which the material (in particular Li) is transported much more slowly than in fully developed convective layers. It has also to be noted that the constraints set in Table 1 by the \(^7\)Li destruction do not allow any significant \(^{11}\)Be depletion at the solar surface.

Of course, it remains to be seen if the predicted deepening of the convective envelope due to the action of the compression effects does not raise other problems relating in particular to stellar evolution or helioseismology. Various tests will be performed in the near future. It can already be remarked that helioseismology can provide accurate values of the convection depth (e.g. Christensen-Dalsgaard et al. 1985). In particular, the use of an inversion sound speed technique leads to an estimate of the extension of the convection envelope amounting to \((0.30 \pm 0.01)R_\odot\). Such a result would exclude in our model high asymmetry between the upward and downward currents in the solar convective cells. In fact, from Table 1, the constraint \( \alpha_1 < 1.8 \) would result if \( \alpha_1 = 1.5 \) and \( a_0 = 1 \).

Another possible remedy, which does not exclude the extended convection mentioned above, is to reduce the amount of \(^7\)Li that has to be destroyed during the Main Sequence phase. This can be achieved if some \(^7\)Li can be burned during the pre-main sequence evolution. If this can indeed happen, Eq. (10) can be satisfied by a less extended transport region. Such a possibility has already been suggested by Bodenheimer (1965), and more recently by e.g. D’Antona & Mazzitelli (1984) (who introduce artificially some pre-main sequence mixing), or Proffit & Michaud (1989). However, this last work still contains some limitations (e.g. in the treatment of the atmosphere). It has to be noted that recent observations by Magazzu & Rebolo (1988) do not exclude such a pre-main sequence Li destruction. However, the accuracy of such observations must be improved. Other studies support this possibility (e.g. Duncan 1981). This question will be examined elsewhere in details and on grounds of improved stellar models.

4. Conclusion

The question of the solar Li and Be surface abundances is reexamined in the light of a new convection model which takes care of the action of compression effects. This is achieved by simulating in a parametric way the asymmetry between the upward and downward flows, and the velocity dispersion around a mean value inside a convective cell. In this way, we are able to reproduce quite satisfactorily the main features of the compressive convection found in 2-D and 3-D hydrodynamical calculations (see Forestini et al. 1990, and references therein).

In a first step, we stress that the extent of the transport zone beneath the convective envelope has to be larger than envisioned

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5 During the main sequence, \( T_b \) never exceeds \( 2.2 \times 10^6 \) K. At such temperatures, the typical \(^7\)Li destruction timescale is about 10 times the duration of this evolutionary phase, so that \(^7\)Li has no time to be depleted further by convection.
in many studies in order to destroy enough $^7$Li during the main sequence phase. This results merely from the proper consideration of the dilution caused by mixing. Severe constraints could ensue on the required efficiency of proposed transport mechanisms, like turbulent diffusion. In a second step, we show that such constraints could be relaxed to some extent if the convective envelope could be deeper than predicted by the standard MLT. Our convection model lends physical support to such a possibility. Of course, many possible consequences of our convection model remain to be explored, and confronted with observations. In particular, the previous and future evolution of the Sun has to be calculated extensively. On the other hand, a pre-main sequence nuclear destruction of Li, if confirmed, can also help solving the problem which could be raised by the required size of the transport region.

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